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MICROWAVE REFLECTION TECHNIQUES FOR DENSE PLASMA DIAGNOSTICS

S. TAKEDA AND T. TSUKISHIMA

U. S. DEPARTMENT OF COMMERCE
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Microwave Reflection Techniques for Dense Plasma Diagnostics

Susumu Takeda* and Takashige Tsukishima

The microwave reflection method for measuring the electron density whose plasma frequency is higher than the probing frequency is described in detail. Various expressions and formulas which are useful for a variety of experimental conditions are given. A method to extend further the measurable density range is proposed. The fringing field effects are considered when a waveguide is used as a probe. References are given to the experimental works which substantiate the theoretical analyses. Also included are the analyses of reflection by a non-uniform boundary and also by an inhomogeneous plasma.

1. Introduction

The determination of the electron density, including its spatial distribution, has been one of the first requisites in plasma researches. Among other methods, microwave techniques have been extensively used because of great simplicity in handling and interpreting the results. The microwave methods hitherto developed may be divided into 1) the cavity method and 2) the propagation method which is applicable to plasmas in a waveguide as well as in free space. The cavity method and the waveguide propagation method are mainly used for fundamental studies of small scale laboratory plasmas, while the free space propagation method has been widely used in various high-temperature devices. A shortcoming of the conventional transmission method is the basic difficulty of investigating the density distribution, as well as a certain cumbersomeness of the arrangements. Another difficulty resides

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in the fact that an electron density whose plasma frequency exceeds the probing frequency is not measurable. Thus, even with 3 mm waves the maximum measurable density is 10^{14}/cc. In order to overcome these difficulties, a new reflection method was proposed by one of the authors.[1] While the conventional reflection method or cutoff method is only for locating plasma,[2] the present method admits measuring an electron density higher than critical. The method is thus different also from that of S. C. Lin et al.,[3] who determined sub-critical densities by measuring reflected power. The principle is based on the fact that the electron density of a uniform plasma bounded by a plane boundary is related, in an extremely simple manner, to the phase angle of the reflected wave at the boundary. Since the interaction between the incident microwave and plasma is limited within a narrow region near the boundary, in other words, the penetration distance is very small when the density is high, the measured value by the present method is essentially local.

The proposed method was first applied successfully to a uniform afterglow plasma produced in an X-band waveguide.[4] Later, an electron density as high as 10^{15}/cc of a shock produced plasma was also measured with the same X-band (9000 Mc/sec.) microwave.[5] A K-band (35 Gc/sec.) microwave reflection probe has been used to obtain the density profile of CANDLE plasma (A magnetically confined argon arc) at the National Bureau of Standards, Washington, D. C. The values obtained by this microwave probe have fitted remarkably with those obtained by other techniques such as the Stark broadening of H_β and Langmuir probe.[6]

In the course of these experiments, a number of formulas and

expression have been derived to account for a variety of boundary conditions. It is the purpose of the present paper to collect these results in one place, although some of them have been published elsewhere. Also included are recent analyses of reflection by a non-uniform boundary and also by an inhomogeneous plasma.

The idea described in Chapter 4 has been proposed by S. Takeda, and the calculations in Chapters 6 and 7 have been made by T. Tsukishima.

2. Principle of Reflection Method

A plane electromagnetic wave, $e^{j(\omega t - k_o \cdot z)}$, propagating along the z-direction in vacuum with angular frequency ω and propagation constant k_o, is reflected by a plane boundary, $z = 0$, separating a homogeneous plasma ($z > o$) and vacuum. The reflection coefficient, R, is given by

$$R = \frac{k_o - k}{k_o + k}, \tag{1}$$

where

$$K = \alpha - j\beta \tag{2}$$

α and β are the phase and attenuation constants in the plasma. They are related to plasma parameters,

$$\begin{aligned} \omega_p^2/\omega^2 &\equiv \eta, \\ \nu/\omega &\equiv \delta, \end{aligned} \tag{3}$$

where ω_p and ω are the plasma frequency and the frequency of the incident wave, ν the collision frequency of electrons in the plasma, by

$$\alpha = \frac{k_o}{\sqrt{2}} \left[\left\{ \left(1 - \frac{\eta}{1+\delta^2}\right)^2 + \left(\frac{\eta\delta}{1+\delta^2}\right)^2 \right\}^{\frac{1}{2}} + \left(1 - \frac{\eta}{1+\delta^2}\right) \right]^{\frac{1}{2}}, \tag{4a}$$

$$\beta = \frac{k_o}{\sqrt{2}} \left[\left\{ \left(1 - \frac{\eta}{1+\delta^2}\right)^2 + \left(\frac{\eta\delta}{1+\delta^2}\right)^2 \right\}^{\frac{1}{2}} - \left(1 - \frac{\eta}{1+\delta^2}\right) \right]^{\frac{1}{2}}, \tag{4b}$$

when there is no magnetic field.

Equations (1), (2), (4a) and (4b) give the complex reflection coefficient R as a function of η and δ. They can be inverted to give η and δ as functions of the modulus, $|R|$, and phase angle, θ, of the reflection coefficient $R = |R|e^{j\theta}$.

For example, let us consider first the case for $\delta = 0$. Eqs. (4a) and (4b) then reduce to

$$\left.\begin{aligned} \alpha &= k_o\sqrt{1-\eta}\,, \\ \beta &= 0, \end{aligned}\right\} \text{ for } \eta < 1, \tag{5a}$$

$$\left.\begin{aligned} \alpha &= 0, \\ \beta &= k_o\sqrt{\eta-1}\,, \end{aligned}\right\} \text{ for } \eta > 1. \tag{5b}$$

Substitution of Eq. (5b) into Eq. (1) gives

$$R = \frac{1+j\sqrt{\eta-1}}{1-j\sqrt{\eta-1}}\,. \tag{6}$$

The phase angle, θ, is immediately obtained from Eq. (6):

$$\theta = 2\tan^{-1}\sqrt{\eta-1}\,,$$

which indicates that θ tends to π as η goes to ∞. Thus, a plasma of infinite density is equivalent to a perfect conductor as far as the reflection coefficient is concerned. It is sometimes convenient to use

$\varphi \equiv \pi - \theta$ rather than θ. Eq. (7) is then written as,

$$\varphi = 2 \tan^{-1} \frac{1}{\sqrt{\eta - 1}} \tag{8}$$

Eq. (8), solved for η, yields,

$$\eta = \frac{1}{\sin^2 \varphi/2} \tag{9a}$$

$$\approx \frac{1}{\tan^2 \frac{\varphi}{2}} \quad \text{for } \eta \gg 1 \tag{9b}$$

The Eq. (9) shows that the normalized density η is readily obtained by measuring the phase angle φ. The value of φ is conventionally calculated from the shift of the standing wave appearing in the vacuum side. The standing wave E is expressed as,

$$E = e^{-jk_o z} + Re^{jk_o z}$$

$$= 2e^{j\frac{\theta}{2}} \cos\left(k_o z + \frac{\theta}{2}\right). \tag{10}$$

Accordingly, the shift of the standing wave minimum, S , as referred to the position for the perfect conductor, is obtained by putting $k_o S + \frac{\theta}{2} = \frac{\pi}{2}$. Then,

$$k_o S = \frac{\varphi}{2} = \tan^{-1} \frac{1}{\sqrt{\eta - 1}} \tag{11}$$

This shift of the standing wave can be expressed in terms of the attenuation or penetration distance, d, defined by

$$d \equiv \frac{1}{\beta}, \tag{12a}$$

$$= \frac{1}{k_o \sqrt{\eta - 1}} \quad \text{for } \eta > 1 \text{ and } \delta = 0 \tag{12b}$$

From Eqs. (8), (11) and (12b), we obtain the following relation:

$$k_o S = \tan^{-1} (k_o d) \tag{13}$$

Thus the shift of the standing wave is equal to the penetration depth of the incident wave into the plasma, provided $k_o d \ll 1$.

For plasmas with $\delta \neq 0$, things are somewhat involved as is seen in Eq. (4). However the expressions for η and δ in terms of $|R|$ and δ have been derived for $\eta \gg 1 + \delta^2$: they are,[1]

$$\eta = \frac{4}{\tan^2\varphi - (1-|R|)^2}, \tag{14a}$$

$$\delta^2 = \left[\frac{1 + (1-|R|)^2/\tan^2\varphi}{1 - (1-|R|)^2/\tan^2\varphi} \right]^2 - 1. \tag{14b}$$

Noting $k_o S = \varphi/2$ and $\tan \frac{\varphi}{2} \cong \frac{1}{2} \tan \varphi$, one can rewrite Eqs. (14a) and (14b) in the following forms,

$$\eta = \frac{1}{\tan^2 k_o S - (1-|R|)^2/4} \tag{15a}$$

$$\delta^2 = \left[\frac{1 + (1-|R|)^2/4 \tan^2 k_o S}{1 - (1-|R|)^2/4 \tan^2 k_o S} \right]^2 - 1. \tag{15b}$$

The values of η against $\tan \varphi$ are shown in Fig. 1 for various values of δ. It is seen that the correction to η due to finite δ is relatively small. For time varying plasmas, it is more convenient to utilize a suitable couple of the standing wave signal. The normalized amplitude, h_z, of the standing wave detector which is assumed to have an ideal square characteristic, is given by,

$$h_z = \frac{1}{4} \left| 1 + R \cdot e^{-j2k_o|z|} \right|^2. \tag{16}$$

The numerical factor 1/4 is introduced in order to normalize all standing wave amplitudes to unit maximum for $R = 1$.

Eq. (16) can be solved for η and δ with two h_z's observed at two different positions. While the choice of these points is to some extent arbitrary, the four points shown in Fig. 2 are preferred to get concise expressions for η and δ. After some calculations and suitable approximations, the following relations are deduced:

a) For $\eta \gg 1 + \delta^2$,

$$\eta = \frac{4}{(h_3-h_4)^2 - \{1-(h_3+h_4)\}^2} \tag{17a}$$

$$\delta^2 = \left\{ \frac{(h_3-h_4)^2 + \{1-(h_3+h_4)\}^2}{(h_3-h_4)^2 - \{1-(h_3+h_4)\}^2} \right\}^2 - 1. \tag{17b}$$

The above equations reduce to

$$\eta \simeq \frac{4}{(h_3-h_4)^2}\ , \tag{18a}$$

$$\delta \simeq \frac{2}{(h_3-h_4)} \cdot \{1-(h_3+h_4)\}, \tag{18b}$$

for $\delta^2 \ll 1$. Eq. (18a) can be slightly modified to give,

$$\frac{\eta^2}{\eta-1} = \frac{4}{(h_3-h_4)^2}\ , \tag{18c}$$

which is still applicable even for $\eta > 1$, provided $\delta \ll 1$.

b) For $\eta > 1$ and $\delta^2 \ll 1$,

$$\eta \simeq 1 + \frac{h_2}{h_1} \tag{19a}$$

$$\delta \simeq \sqrt{\frac{h_2}{h_1}} \cdot \{1 - (h_1+h_2)\} \tag{19b}$$

In deriving Eqs. (19a) and (19b), α and β have been approximated by

$$\alpha \simeq k_o \eta \delta / 2\sqrt{\eta-1} \tag{20a}$$

$$\beta \simeq k_o \sqrt{\eta-1} \tag{20b}$$

Eqs. (18c) and (19a) simplify respectively to

$$\frac{\eta^2}{\eta-1} = \frac{1}{(1/2-h_4)^2} = \frac{1}{(h_3-1/2)^2} \tag{21a}$$

and

$$\eta = \frac{1}{h_1} \tag{21b}$$

when $\delta = 0$. Eq. (21b) will be seen to be exactly equivalent to Eq. (9a).

Since the phase and attenuation constants in a plasma filled waveguide through which the microwave excited to TE_{10} mode propagates can be obtained by merely replacing η in Eqs. (4a) and (4b) by $\eta/(1-\lambda^2/\lambda^2_c)$ where λ and λ_c are the wavelength in free space and the cutoff wavelength of the waveguide, the formulas in this chapter and in the following chapters can be applied for a waveguide system provided η is replaced by $\eta/(1-\lambda^2/\lambda^2_c)$. Eqs. (19a) and (19b) have been successfully used to determine η and δ in a recombination controlled afterglow plasma which was produced in an X-band rectangular waveguide. In this experiment an electron density as high as 10^{13}/cc has been measured.[4]

3. Some Considerations for Reducing Experimental Errors

3.1 Difference Method

In order to be able to measure a density as high as $\eta = 10^4$ one has to measure within a reasonable error range a value of (h_3-h_4) as small as 2×10^{-2} or 1.1 degree in the phase angle. $\eta = 10^4$ also corresponds to the shift of the standing wave minimum of $S = (\lambda/2\pi) \times 10^{-2}$ cm $\simeq 5\times10^{-3}$ cm for an X-band microwave of $\lambda = 3$ cm. Thus the correct location of the standing wave detectors is of the primary importance.

However, the errors in η and δ due to the slight mis-location of the detectors can be to some extent avoided by a sort of difference method when it is possible to get an electron density much higher than those which are to be measured. Such a condition as above is often fulfilled in shock experiments.

Let h_3' and h_4' be the standing wave amplitudes for a plasma whose density is much higher than those of interest; then η and δ can be expressed in terms of

$$\begin{aligned} \Delta h_3 &\equiv h_3 - h_3', \\ \Delta h_4 &\equiv h_4 - h_4', \end{aligned} \tag{22}$$

as follows;

$$\eta = \frac{4}{(\Delta h_3 - \Delta h_4)^2 - (\Delta h_3 + \Delta h_4)^2}, \tag{23a}$$

$$\delta^2 = \left\{ \frac{(\Delta h_3 - \Delta h_4)^2 + (\Delta h_3 + \Delta h_4)^2}{(\Delta h_3 - \Delta h_4)^2 - (\Delta h_3 + \Delta h_4)^2} \right\}^2 - 1, \tag{23b}$$

provided $\delta/\sqrt{\eta} \gg \delta'/\sqrt{\eta'}$ and $(2\pi/\lambda)(\Delta\chi_3 + \Delta\chi_4) \ll 1$, where $\Delta\chi_3$ and $\Delta\chi_4$ are the errors in the positions of the standing wave detectors for h_3 and h_4 respectively, and δ' and η' are the values for the reference high density plasma.

Eqs. (23a) and (23b) reduce to

$$\eta \approx \frac{4}{(\Delta h_3 - \Delta h_4)^2}, \tag{24a}$$

$$\delta \approx 2 \cdot \frac{\Delta h_3 + \Delta h_4}{\Delta h_3 - \Delta h_4} \tag{24b}$$

for $\delta^2 \ll 1$. Note that $(\Delta h_3+\Delta h_4) < (\Delta h_3-\Delta h_4)$ because of $\Delta h_3 > 0$ and $\Delta h_4 < 0$.

Eq. (24a) has been used to evaluate the electron densities from 10^{13} to 3×10^{15}/cc in a plasma produced by an electromagnetically driven shock wave into argon.[5]

3.2 Effect of Power Losses Along the Path Between the Plasma Boundary and the Standing Wave Detectors

The power losses, such as the circuit loss due to the finite conductivity of the waveguide or some scattering loss between a horn antenna and the plasma boundary, have to be taken into account to get the correct values of η and δ. When these losses are of pure dissipative type and accordingly give rise to no phase shift, the standing wave signal, h_z, given by Eq. (16) is modified as follows:

$$h_z = \frac{1}{4}\left|1 + \rho Re^{-j2k_o|z|}\right|^2 . \tag{25}$$

Here, a correction factor, ρ, which is a real number less than one, is introduced to account for these power losses. The similar calculations as in the preceding chapter result in,

$$\eta \approx \rho^2 \frac{4}{(h_3-h_4)^2} , \tag{26a}$$

$$\delta \approx \frac{\rho}{h_3-h_4} \cdot \left[1 + \frac{1}{\rho^2}\{1 - 2(h_3+h_4)\}\right], \tag{26b}$$

provided $\eta > 1$ and $\delta^2 \ll 1$.

Alternate expressions for η and δ in terms of h_1 and h_2 are possible: they are,

$$\eta = 1 + h_{20}/h_{10}, \tag{27a}$$

$$\delta = \frac{h_{20}}{h_{10}} \cdot \{1 - (h_{10}+h_{20})\}, \tag{27b}$$

with

$$\begin{aligned} h_{10} &= \frac{1}{2\rho} \{\sqrt{2(h_1+h_2)-1} - (h_2-h_1)\}, \\ h_{20} &= \frac{1}{2\rho} \{\sqrt{2(h_1+h_2)-1} + (h_2-h_1)\}. \end{aligned} \tag{28}$$

When $\delta = 0$, Eq. (27a) reduces to

$$\eta = \frac{\rho}{h_1 - \frac{1}{4}(1-\rho)^2}, \tag{29}$$

by noting that $h_1+h_2 = \frac{1}{2}(1+\rho^2)$ for $\delta = 0$.

Eqs. (26a) and (26b) can again be rewritten in terms of Δh_3 and Δh_4 defined in Eq. (22);

$$\eta = \rho^2 \cdot \frac{4}{(\Delta h_3 - \Delta h_4)^2} \tag{30a}$$

$$\delta = \frac{2}{\rho} \cdot \frac{\Delta h_3 + \Delta h_4}{\Delta h_3 - \Delta h_4} \tag{30b}$$

Eqs. (26), (29) and (30) reduce to Eqs. (18), 21 b and (24) respectively when $\rho = 1$.

Note that the introduction of the correction factor ρ does not result in any change in the shift of the standing wave minimum S, as is clear from Eq. (25) and the definition of ρ.

4. Further Extension of Measurable Density Range

As is well known in transmission theory, a small phase shift can be expanded by making use of a suitable buffer dielectric plate of $\lambda_\epsilon/4$

thickness. This idea is here utilized to extend further the measurable density range. The reflection coefficient R at the surface of the dielectric plate, whose other side is in contact with plasma as is shown in Fig. 3, can be easily calculated to give,

$$R = \frac{k_3/k_o - (k_2/k_o)^2}{k_3/k_o + (k_2/k_o)^2} \tag{31}$$

where

$$\begin{aligned} k_3 &= -jk_o\sqrt{\eta-1} \\ k_2 &= k_o\sqrt{\varepsilon} \end{aligned} \tag{32}$$

and ε is the specific dielectric constant of the plate.

Substitution of Eq. (32) into (31) gives

$$R_1 = -\frac{1 + j\sqrt{\eta-1}/\varepsilon}{1 - j\sqrt{\eta-1}/\varepsilon}, \tag{33}$$

which, when re-evaluated at a point off the dielectric boundary by a distance $\lambda/4$, turns out to be

$$R_1 = \frac{1 + j\sqrt{\eta-1}/\varepsilon}{1 - j\sqrt{\eta-1}/\varepsilon} \tag{34}$$

The shift of the standing wave minimum, S_1, is now given by

$$\begin{aligned} k_o S_1 &= \tan^{-1}\cdot\frac{\varepsilon}{\sqrt{\eta-1}} \\ &\approx \frac{\varepsilon}{\sqrt{\eta}} \quad \text{for } \eta \gg 1 \end{aligned} \tag{35}$$

Eq. (35) shows that S_1 is nearly ε times S of Eq. (11). Accordingly the maximum, measurable value of η is risen up by a factor ε^2. Eq. (35) can be easily generalized to the case where n dielectric plates are placed in front of the plasma boundary, each plate being separated by a distance

$\lambda/4$ from its neighbors. Then the shift of the standing wave minimum, S_n, observed in front of the n-th plate is given by

$$k_o S_n = \tan^{-1} \frac{\varepsilon^n}{\sqrt{\eta - 1}} . \tag{36}$$

It is noted that the first dielectric plate does not necessarily have to be in contact with the plasma; the plate located at distance $\lambda/2$ from the plasma boundary gives the same result.

The technique described here can be also applied to plasmas whose densities are less than critical. When the plasma shown in Fig. 3 is replaced by a plasma layer of thickness L, of density $\eta < 1$ and whose other boundary is made in contact with a perfect conductor, the relection coefficient evaluated at a distance $\lambda/4$ off the dielectric plate is given by

$$R_1' = \frac{1 + j\sqrt{1-\eta}/\varepsilon \cdot \tan k_o L\sqrt{1-\eta}}{1 - j\sqrt{1-\eta}/\varepsilon \cdot \tan k_o L\sqrt{1-\eta}} . \tag{37}$$

If we choose $k_o L = 2m\pi$ where m is an interger, we have $\tan k_o L \sqrt{1-\eta} \cong m\pi\eta$ for $\eta \ll 1$. Thus the shift of the standing wave minimum, S_1', is given by

$$k_o S_1' \approx m\pi\varepsilon\eta, \tag{38}$$

which is ε times larger than that without the dielectric plate. The minimum, measurable value of η is therefore lowered by a factor ε. With n plates, being separated by a distance $\lambda/4$ from each other, the shift of minimum observed in front of the n-th plate is,

$$k_o S_n' = m\pi\varepsilon^n\eta. \tag{39}$$

The other boundary of the plasma layer needs not necessarily be in contact with the metallic conductor; the standing wave can be constructed from two signals in a bridge type circuit. One signal is taken directly

from the microwave oscillator, while the other is taken through the plasma layer. Thus the present technique covers a wide range of measurable density, for instance, from $\eta = 10^{-4}$ to 10^{6} when two dielectric plates of $\epsilon = 3.7$ are used for a plasma layer whose thickness is $\lambda/2$. In a waveguide, ϵ has also to be replaced by $(\epsilon-\lambda^2/\lambda_c^2)/(1-\lambda^2/\lambda_c^2)$.

5. Microwave Reflection Probe

Hitherto we have been mainly concerned about plane wave analyses. However additional problems arise when a microwave probe, as is illustrated in Fig. 4, is used to obtain the local electron density of a plasma in free space.

First of all, a part of incident microwave power is reflected at the probe end even when there is no plasma outside the probe, because of the abrupt change of geometry. Of course this type of reflection can be eliminated by using a suitable matching device such as three stubs or E-H tuner. The problem is whether the tuner, once matched for free space, has to be re-adjusted when the probe is inserted into plasmas. An analysis has been made of the equivalent circuit based on the theory of the transmission lines; the result shows that the overall reflection coefficient is given by a product of a constant phase factor and the reflection coefficient of the plasma boundary, provided the system is matched initially for free space. This constant phase factor can experimentally be corrected for by referring phase to the metallic short case after the matching for free space is achieved.

The second and more serious problem is how to handle the distortion of the electric field outside the probe. Galejs has calculated the

admittance of a waveguide radiating into plasma,[7] and Wait has given a formal solution to the radiation field outside a slotted conducting plane in a plasma environment.[8] In both cases, the electromagnetic wave propagates into the plasma through a slot antenna which is located in a perfectly conducting infinite plane.

Instead of having recourse to the rigorous, but cumbersome mathematics, we develop here an heuristic and semi-empirical method.[9] We first assume that the fringing field in vacuum can be expressed approximately by the field produced by a line dipole placed at distance Z_o behind the probe end. Then the field strength at distance z from the probe end is given by $E_o Z_o^2/(z+Z_o)^2$. This has to be multiplied by the attenuation factor exp $(-z/d)$ in order to give the field when plasma is present. Thus, to a first approximation, we obtain

$$E(z) = E_o\left(\frac{Z_o}{z+Z_o}\right)^2 \cdot \exp\left(-\frac{z}{d}\right) \tag{40}$$

for the fringing field in plasma, where E_o is the value at $z = 0$, and Z_o depends presumably on the ratio of the wavelength to the characteristic probe dimension. On the other hand, we can also express $E(z)$ formally in terms of an effective penetration depth, d', as follows:

$$E(z) = E_o \exp\left(-\frac{z}{d'}\right). \tag{41}$$

To define d' in terms of d and Z_o, the two expressions for $E(z)$ are set equal at some point $z = Z_1$. While the choice of such a point seems arbitrary, we may argue on physical grounds that the best choice of Z_1 would be $Z_1 = d$, where the field is roughly e^{-1} times E_o. The result is:

$$d' = d[1 + 2\ell n(1+d/Z_o)]^{-1}. \qquad (42)$$

This equation reduces to the correct limit, $d'/d = 1$, for large η or small d, for it is just when $d \ll Z_o$ that fringing field effects are expected to be negligible. For d large, $d'/d < 1$ expresses the fact that a smaller effective penetration depth is here used to compensate for the field decrease caused by the effect of geometry. With Eq. (42), one can easily obtain the reflection coefficient, noting $d' = 1/\beta'$ and $d = 1/\beta$. Or one may make use of Eq. (12b) to compare η' with η, where η is the actual density just outside the probe and η' includes the fringing field effect. In Fig. 5 are shown the values of d'/d and η'/η against d/Z_o for various values of Z_o/λ. The value of Z_o remains to be determined experimentally. And this was done with a uniform argon afterglow plasma produced in a cylindrical waveguide. The phase shift was simultaneously measured with an X-band microwave at the two ends of the chamber. At one end, the chamber was attached to a cylindrical waveguide of the same diameter as the discharge tube so that fringing fields are absent. The microwave probe was used at the other end. It consisted of a standard RG-52/U waveguide tapered to a 3 mm height so that the plasma in the cylindrical chamber is considered as a plasma in free space for the probe. The calculated and observed values of η' showed good agreement within the experimental range of $\eta < 10$ when Z_o/λ_g was chosen equal to 0.63.

A K-band microwave probe of the cross section 7 x 1 mm^2 was successfully used to determine the density profile of an argon arc plasma. The

densities measured by the probe fitted quite well with those obtained by other techniques such as the Stark broadening of Hβ line and Langmuir probe.[6]

6. Reflection by Inhomogeneous Plasma

Need sometimes arises to take account of inhomogeneity of the electron density along the direction normal to the boundary surface, when one wants, for instance, to measure the density profile of a cylindrical plasma whose characteristic diameter is of a comparable order of magnitude as the wavelength of the probing signal.

To choose a simple case for illustration, let us consider a linear density ramp with a sharp cut at $z = 0$ where the reflection probe is located, as is depicted in Fig. 6. Since the gradient of the electron density is parallel to the direction of the propagation of the incident wave, the wave equation for the electric field strength, E, is simply given by[10]

$$\frac{d^2E}{dz^2} + k_o^2(1-\eta)\, E = 0, \tag{43}$$

with

$$\begin{aligned} \eta &= 0 \qquad\quad \text{for } z < 0, \\ \eta &= \eta_o + \frac{z}{Z_o} \quad \text{for } z \geq 0. \end{aligned} \tag{44}$$

The solutions for the region I and II, in Fig. 6, is given by

$$E_I = e^{-jk_o z} + R\, e^{jk_o z}, \tag{45}$$

$$E_{II} = A\sqrt{\chi}\, I_{1/3}(\xi) + B\sqrt{\chi}\, I_{-1/3}(\xi), \tag{46}$$

where

$$\chi \equiv \eta_1 - 1, \tag{47a}$$

$$\xi \equiv \frac{2}{3} k_o Z_o \chi^{3/2}, \tag{47b}$$

and $I_{\pm 1/3}(\xi)$ are the modified Bessel functions of orders $\pm 1/3$. The boundary condition that E_{II} has to vanish at $z \to \infty$ requires $A = -B$, since the asymptotic form of $I_{\pm 1/3}(\xi)$ is given by[11]

$$I_{\pm 1/3}(\xi) \xrightarrow[\xi \to \infty]{} (2\pi\xi)^{-1/2} \left[e^{\xi} + e^{-\xi} \, e^{-j(\frac{1}{2} \pm \frac{1}{3})\pi} \right]. \tag{48}$$

The reflection coefficient R is obtained from Eqs. (45) and (46) by putting $E_I = E_{II}$ and $\frac{dE_I}{dz} = \frac{dE_{II}}{dz}$ at $z = 0$.

After some calculations, we obtain

$$R = \frac{1-jK}{1+jK} \tag{49a}$$

where

$$K = \left[\frac{\chi I_{-2/3}(\xi) + \chi I_{2/3}(\xi)}{\sqrt{\chi} I_{1/3}(\xi) - \sqrt{\chi} I_{-1/3}(\xi)} \right] \quad z = 0. \tag{49b}$$

When $k_o Z_o (\eta_o - 1)^{3/2} \gg 1$, in other words, when the density gradient is very small and/or η_o is very high, Eq. (49b) reduces to

$$K \approx -\sqrt{\eta_o - 1} \left\{ 1 + \frac{1}{4k_o Z_o} \cdot \frac{1}{(\eta_o - 1)^{3/2}} \right\}. \tag{50}$$

Hence, we have

$$R \cong \frac{1 + j\sqrt{\eta_o - 1} \left\{ 1 + \frac{1}{4k_o Z_o} \cdot \frac{1}{(\eta_o - 1)^{3/2}} \right\}}{1 - j\sqrt{\eta_o - 1} \left\{ 1 + \frac{1}{4k_o Z_o} \cdot \frac{1}{(\eta_o - 1)^{3/2}} \right\}}. \tag{51}$$

As is expected, Eq. (51) reduces to Eq. (6) when $Z_o \to \infty$, that is, when the density gradient approaches zero.

It can be shown that a WKB-type approximation method leads to the same result as Eq. (51). When the change in the electron density over a range considered is very small compared to the value at the boundary, the electric field can be approximated by

$$E_{II} \approx C\, e^{-k_o \int_o^z \sqrt{\eta-1}\, dz}, \tag{52}$$

where C is a constant. The effective attenuation distance, d, is then defined by

$$k_o \int_o^d \sqrt{\eta-1} \cdot dz = 1. \tag{53}$$

The integrand can be expanded as follows,

$$\sqrt{\eta-1} = \sqrt{\eta_o + \frac{z}{Z_o} - 1}$$

$$\approx \sqrt{\eta_o-1}\left\{1 + \frac{z}{2Z_o(\eta_o-1)}\right\}. \tag{54}$$

Inserting Eq. (54) into Eq. (53) and solving for d, we obtain,

$$\frac{1}{d} \approx k_o \sqrt{\eta_o-1}\left\{1 + \frac{1}{4k_oZ_o} \cdot \frac{1}{(\eta_o-1)^{3/2}}\right\}, \tag{55}$$

provided $4k_oZ_o(\eta_o-1)^{3/2} \gg 1$. Since $d = 1/\beta$ and $R = (k_o+j\beta)/(k_o-j\beta)$ for a loss-less plasma, Eq. (55) gives the same reflection coefficient as Eq. (51).

7. Reflection by Boundary Layer with Density Gradient

In the preceding chapters, it has been assumed that the reflection always takes place at a well defined boundary surface across which the electron density changes stepwise. However, there exists always a finite boundary layer in an actual plasma, at least, of the order of the Debye length. The boundary layer may become still thicker due to diffusion into the waveguide, when a waveguide probe is inserted into a plasma and when it is impossible, due to heat damage or charge accumulation which might cause some undesirable effect on the plasma, to use a thin insulator to divide the plasma and vacuum.

Microwave reflection by a non-uniform boundary has been treated by many authors: Taylor has calculated reflections at linear density ramps[12] and also at an inverse parabolic density profile,[13] Wait has given a formal solution for a stratified plasma,[14] Albini and Jahn have made numerical calculations of reflection and transmission coefficients for linear and "kinked" ramps, connecting uniform plasmas and for trapezoid geometries,[10] and recently an exponential electron distribution has been attacked by Yen.[15]

Our problem here is to seek the phase shift of the reflection coefficient caused by the existence of a relatively thin boundary layer. Jahn has already given an answer for a special case. He has derived the following formula for the geometry shown in Fig. 7,[16]

$$R_J = \frac{\Omega_2^* + \frac{k_o}{2k_2^*}\left[(1-\Omega_2^*)^2 - 1\right] + \frac{j\Omega_2}{k_o z_1}\left(1 - \frac{k_o}{k_2^*}\right) + \frac{jk_o z_1}{2\Omega_2^*}\left[1 - \Omega_2^* - \frac{k_o}{k_2^*}(1-\Omega_2^*)^2\right]}{\Omega_2^* + \frac{k_o}{2k_2^*}\left[1-(1-\Omega_2^*)^2\right] - \frac{j\Omega_2}{k_o z_1}\left(1 + \frac{k_o}{k_2^*}\right) - \frac{jk_o z_1}{2\Omega_2^*}\left[1 - \Omega_2^* + \frac{k_o}{k_2^*}(1-\Omega_2^*)^2\right]}, \quad (56)$$

where

$$\Omega_2^* = \frac{\omega_\rho^2/\omega^2}{1+\nu^2/\omega^2}\cdot\left(1+j\,\frac{\nu}{\omega}\right)$$
$$k_2^* = \alpha - j\beta. \quad (57)$$

In deriving Eq. (56), the Bessel functions of the forms $J_{\pm 1/3}(\xi)$ have been approximated by the first few terms of their series expansions.

Jahn has also shown that, for a special case of $\eta = 1$ and $\delta = 0$, R_J turns out to be

$$R_J = \frac{1 - j\cdot\frac{1}{2}k_o z_1}{1 + j\cdot\frac{1}{2}k_o z_1}, \quad (58)$$

provided $k_o z_1 \ll 1$. We see from Eq. (58) that the boundary layer with linear ramp over z_1 is equivalent to a step boundary located at $z = \frac{z_1}{2}$ as far as the reflection is concerned.

Although Eq. (56) was obtained for $\eta < 1$, it can be shown that Eq. (56) is also valid for $\eta > 1$. Accordingly the statement made under Eq. (58) can be generalized for $\eta > 1$. To show this, we replace Ω_2^* and k_2^* in Eq. (56) by η and $-jk_o\sqrt{\eta-1}$ respectively, assuming $\delta = 0$, then

$$R_J = \frac{A + jB}{A - jB}, \quad (59)$$

with

$$A = \eta + \eta\sqrt{\eta-1}\cdot k_o z_1 + \frac{(1-\eta)^2}{2\eta}\cdot(k_o z_1)^2, \quad (60a)$$

$$B = \eta\sqrt{\eta-1} + \frac{1}{2}(\eta-2)\, k_o z_1 + \frac{1-\eta}{2\eta} \cdot \sqrt{\eta-1} \cdot (k_o z_1)^2. \tag{60b}$$

The third term in Eqs. (60a) and (60b) can be neglected when $k_o z_1 \ll 1$. Hence we have

$$R_J = \frac{1 + \sqrt{\eta-1} \cdot k_o z_1 + j\left\{\sqrt{\eta-1} + \left(\frac{\eta}{2} - 1\right) k_o z_1\right\}}{1 + \sqrt{\eta-1} \cdot k_o z_1 - j\left\{\sqrt{\eta-1} + \left(\frac{\eta}{2} - 1\right) k_o z_1\right\}} \tag{61}$$

The Eq. (61) divided by Eq. (6) yields,

$$\frac{R_J}{R} = \frac{1 + \frac{k_o z_1}{2}\sqrt{\eta-1} - j\,\frac{k_o z_1}{z}}{1 + \frac{k_o z_1}{2} \cdot \sqrt{\eta-1} + j\,\frac{k_o z_1}{2}} \tag{62}$$

which reduces to

$$R_J/R \cong e^{-jk_o z_1}, \tag{63}$$

when

$$k_o z_1 \sqrt{\eta-1} \ll 1. \tag{64}$$

Eq. (63) implies that a boundary layer with linear ramp over z_1 connecting homogeneous plasma of density η is equivalent to a step boundary located at $z = z_1/2$ of a uniform plasma having the same density.

8. Concluding Remarks

One of the advantages of the reflection method lies in the fact that the ratio of the sheath thickness, which is supposed to be of the order of the Debye length λ_D, to the penetration distance d does not depend on the electron density, but only a function of the electron temperature T_e: since $\lambda_D \cong v_T/\omega_p$ and $d \cong c/\omega_p$, we have $\lambda_D/d \cong v_T/c$, where c and v_T are the light velocity and the mean thermal velocity of

electrons. Thus, the thickness of the boundary layer can always be neglected compared to the penetration distance provided $v_T/c \ll 1$.

Although we have mainly dealt with plasmas in the absence of a magnetic field, it is clear that the formulas can be used as they stand for magnetized plasmas if geometry is chosen such that $\underset{\sim}{k} \perp \underset{\sim}{B}$ and $\underset{\sim}{E} \parallel \underset{\sim}{B}$, where $\underset{\sim}{B}$ is the magnetic field.

When the extraordinary mode of propagation is utilized, that is, when $\underset{\sim}{k} \perp \underset{\sim}{B}$ and $\underset{\sim}{E} \perp \underset{\sim}{B}$, the condition $\omega_c^2 \ll \omega_p^2$ has to be satisfied.

The argument described in Chapter 6 suggests that a waveguide probe without insulating film at its end could be used for a diffusion experiment: the diffusion coefficient could be calculated from the diffusion length at the probe end.

9. Acknowledgment

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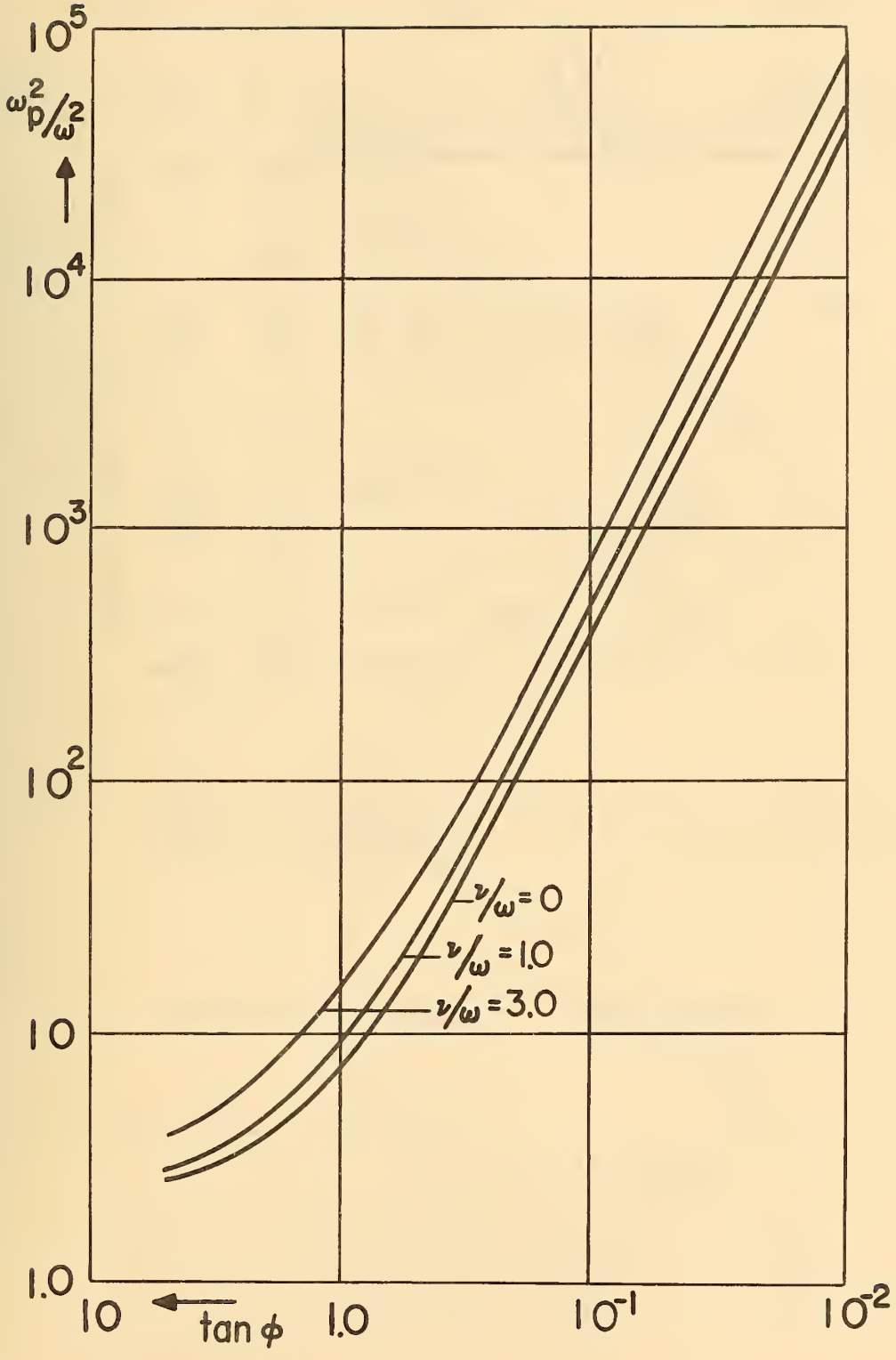

Fig. 1. Relation between η and tan φ for various values of δ.

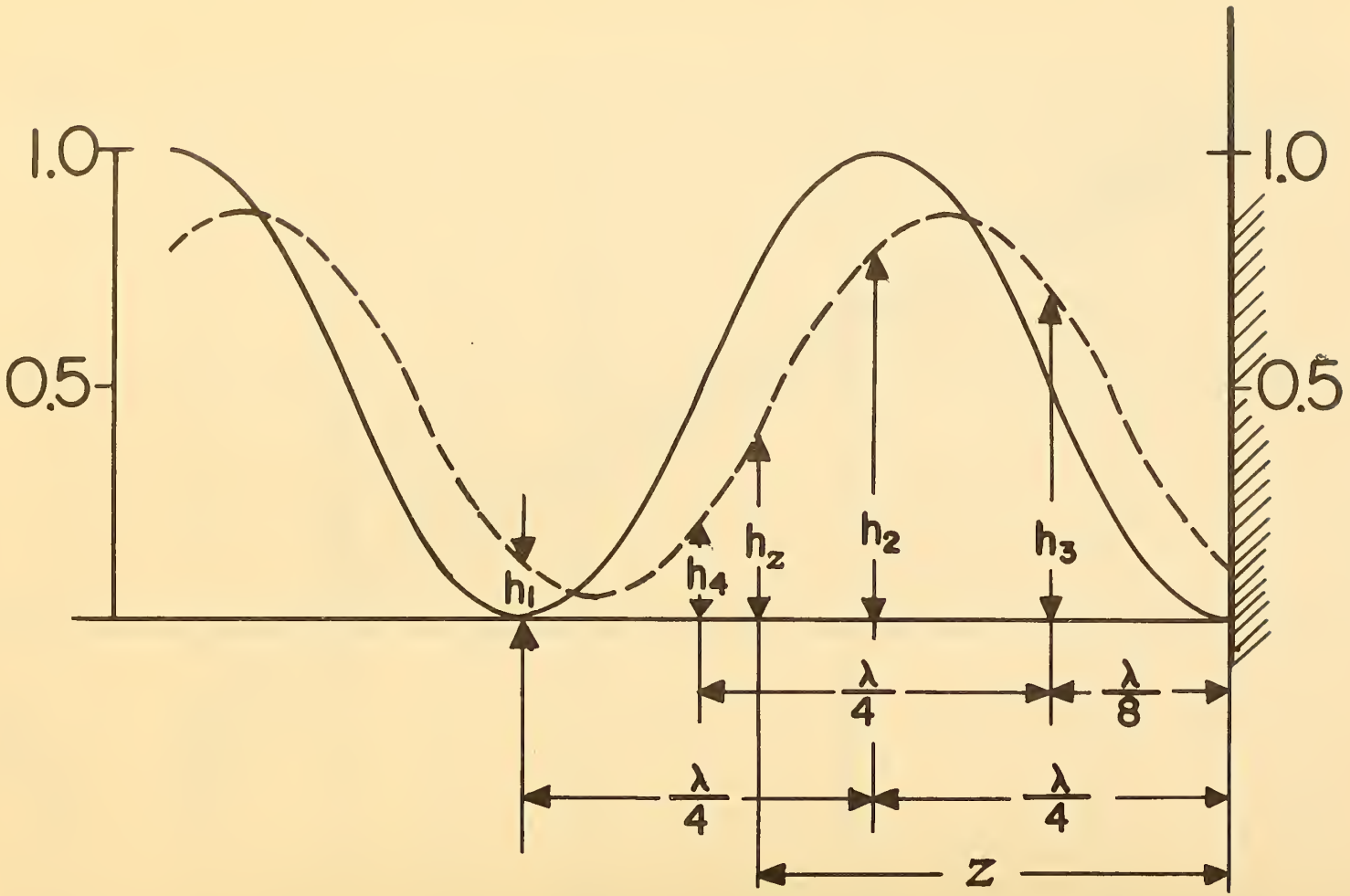

Fig. 2. Squared amplitude of standing wave in front of plasma boundary. The solid line shows the standing wave for a perfect conductor and the dotted line shows that for plasma.

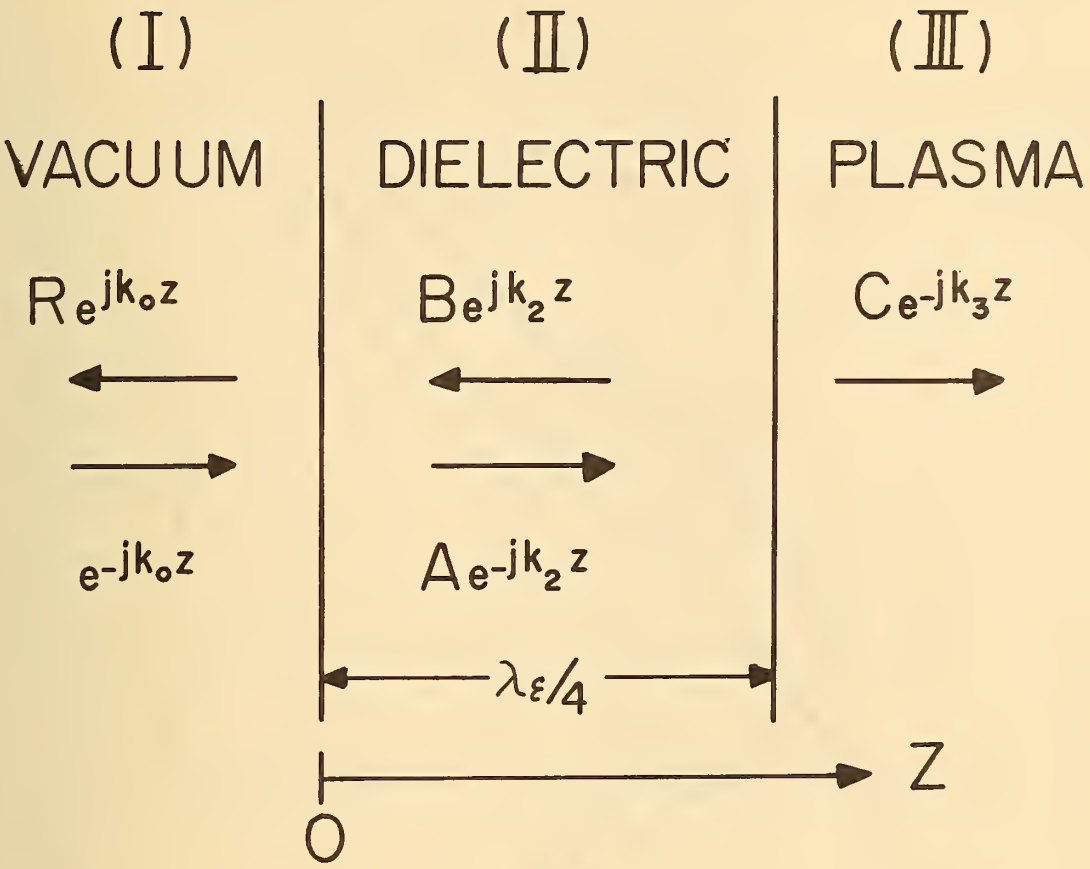

Fig. 3. Reflection by plasma through a dielectric plate whose thickness is $\lambda_\varepsilon/4$, where λ_ε is the wavelength in the dielectric.

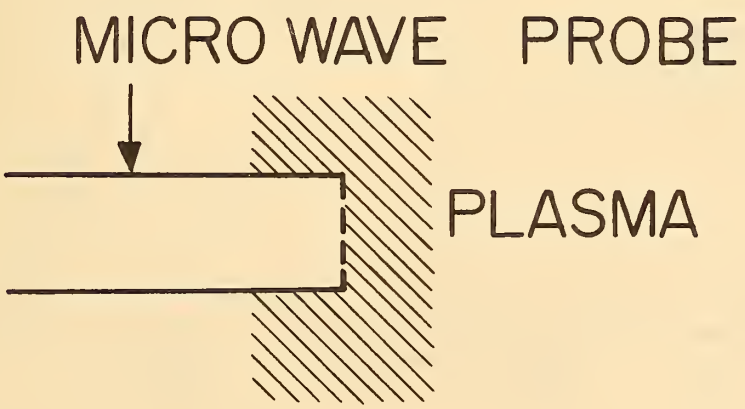

Fig. 4. Sketch of microwave probe.

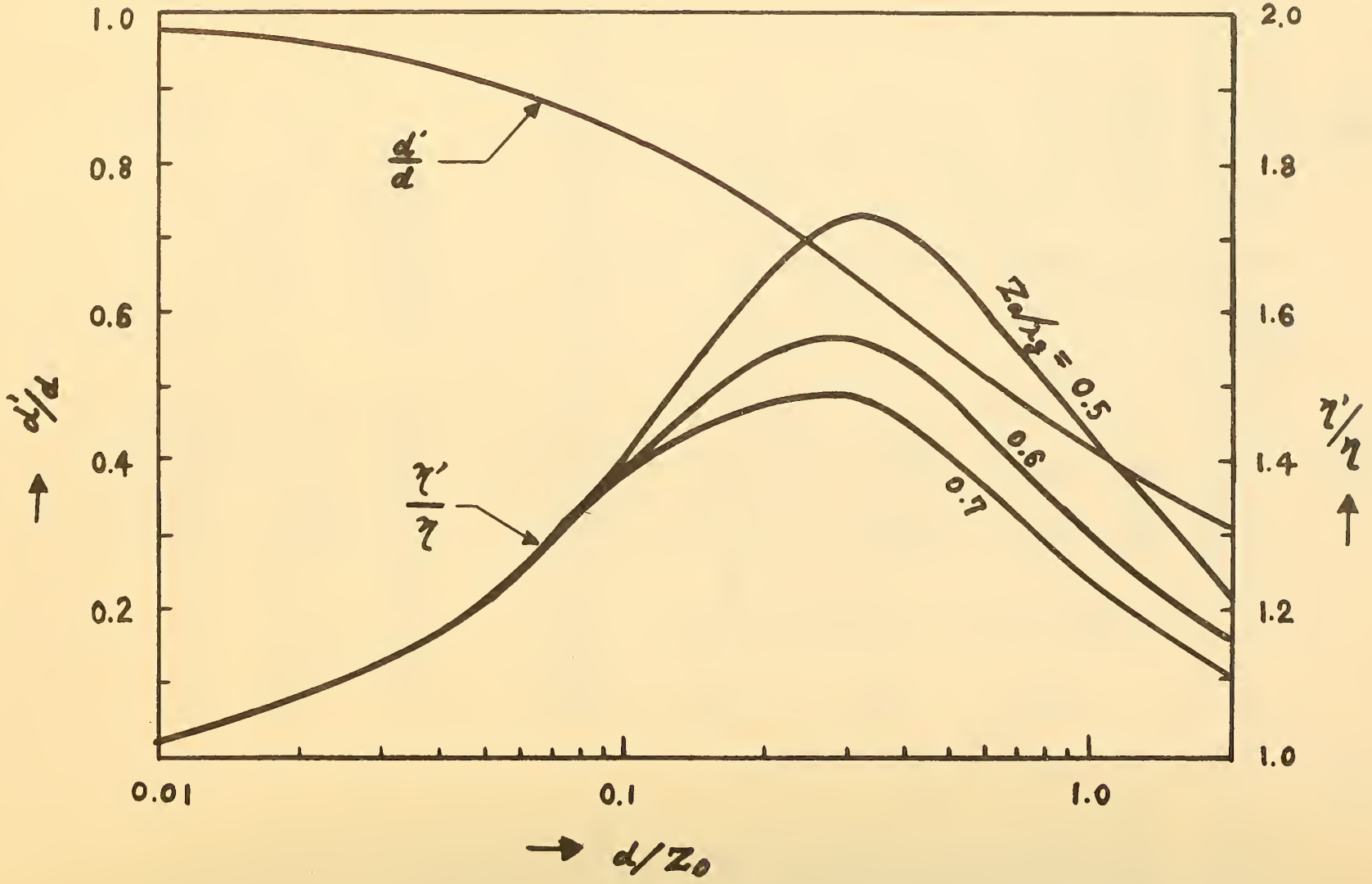

Fig. 5. The values of d'/d and η'/η as functions of d/Z_o.

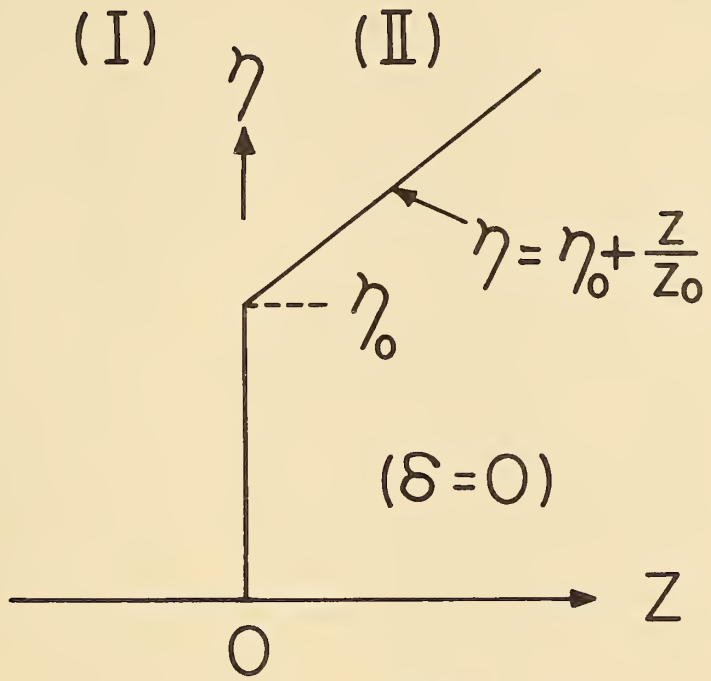

Fig. 6. Density profile of the inhomogeneous plasma having a linear ramp.

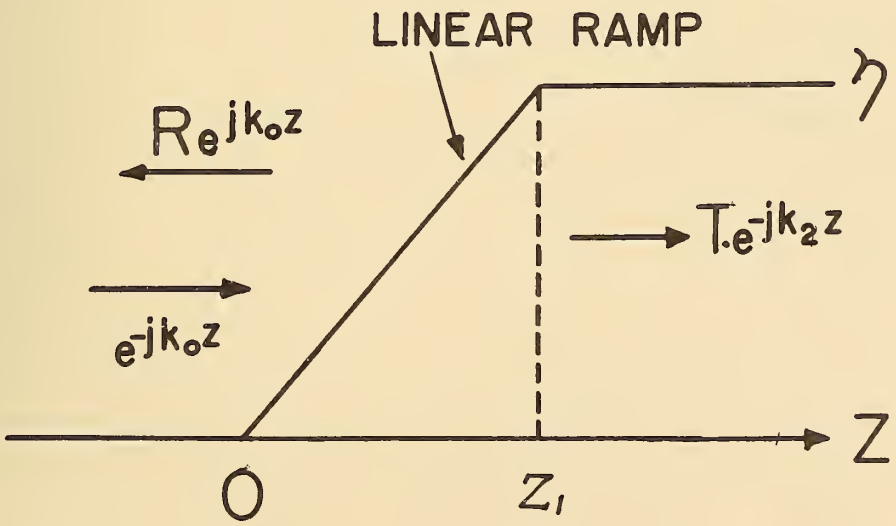

Fig. 7. Density profile of the boundary layer with a constant density gradient.

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